§13.1. ***Synopsis: Chapter Thirteen.***

This chapter is among the more enigmatic and mathematically forward-looking of Briggs' published work. We have seen in the previous two chapters the use of two methods of sub-tabulation: initially, simple proportion is tried with some success, and then what is now known as the second order forward divided difference method is used for sub-tabulating the logarithms of the first 20,000 numbers, and subsequently from 90,000 to 100,000; however, outside these limits the method fails due to differences of higher orders than the second being encountered.

Now Briggs had found a method that involved correcting what he termed divided differences, suitable for all orders, with which to sub-tabulate the rest of the tables from 20,000 to 89,000. The method is presented mainly for quinquisection, with a little said about trisection, and Briggs notes the generality of the method by applying it to tables of powers of integers, sines, etc. Differences of a given order are corrected by subtracting simple sums of multiples of already correct or corrected differences of higher orders. No hint is given by Briggs of how he came upon the method: though he mentions elsewhere that it originated with his work on tables of sines in the early 1600's. A modern derivation of the method is given in the Notes to the chapter, following Goldstine, and we speculate on how Briggs himself might arrived at the method.

§13.2. *Chapter Thirteen*. [p.27.]

It is desired to find the logarithms of a Chiliad. Or, for as many equally spaced given numbers as please, together with the logarithms of these; for the individual intervals of these four numbers, to find the logarithms of the intermediate numbers .

The logarithms of intermediate numbers can be calculated in many ways. I especially recommend the present method to be held in esteem; and we shall see about the rest afterwards.

The first, second, third, fourth, fifth, etc. differences of the given logarithms are taken; and the first difference is divided by 5, the second by 25, the third by 125, etc ; with the ratio of the division increasing in the quintuplet ratio: the quotients are called the first, second, third, &c, mean differences; or rather in place of division, it can be done by multiplication of the first given difference by 2, of the second by 4, the third by 8, etc, cutting off one figure in products with the first order, two with the second, three with the third, etc The products which are equal to those quotients are the first, second , third , etc mean differences. For let the given logarithms [Table 13-1] together with the first, second, third, fourth, and fifth differences of these be given, which the given logarithms themselves show by subtraction:

fifth	*fourth*	*third*	*second*	*first*	*logarithms*	*Absolute numbers*
		1151695		102791641337		
	8138		242719568		33253,10371,71106	2115
75		1143557		102548921769		
	8063		241576011		33263,35860,92875	2120
75		1135494		102307345758		
	7988		240440517		33273,58934,38633	2125
		1127506		102066905241		

[Table 13-1]

The required mean differences are placed nearby [Table 13-2], by multiplication of the first differences given by 2, the first with one figure taken away from the products. The remaining means are constructed if the given difference numbers are multiplied by 4, 8, 16, 32, etc.

These means are then to be corrected in this way[1]:

The two most distant means, the fourth and fifth, of course, do not need correcting (because the sixth and seventh differences are zero; but for all the other differences, the correction are made by taking away other more removed and correct differences: as the subtraction of the seventh corrects the fifth: of the sixth the fourth, etc). The fourth and fifth means are therefore taken as the fourth and fifth corrected means; while the third means are corrected, if from the same are taken away three times the fifth corrected means.

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	Mean differences	
	20558328267|4	
	20509784353|8	by 2
first	20461469151|6	
	2041338104|82	
	9708782|72	
sec.	9663040|44	by 4
	9617620|68	
	9213|560	
third	9148|456	
	9083|952	by 8
	9020|048	
	13|0208	
fourth	12|9008	by 16
	12|7808	
fifth	|02400	by 32
	|02400	

	9213	560 third mean
D		72 3 times fifth correction
	9213	488 third correct mean
	9148	456 third mean
		72 3 times fifth correction
	9148	384 third correct mean
	9083	952 third mean
		72 3 times fifth correction
	9083	880 third correct mean
	9020	048 third mean
		72 3 times fifth correction
	9019	976 third correct mean

From the second means are taken away double the fourth correct means, in addition $1\frac{2}{5}$ of six order is taken away, if any of the sixth [order] are found between these limits.