

§26.1

Chapter Twenty - Six: Synopsis.

This is a long chapter concerned with surface and volume calculations of the ellipse and spheroid, and sections of the spheroid, according to the formulae of Archimedes; finally, the methods developed are applied to cask gauging, a popular mathematical pursuit at the time.

§1. For a given ellipse with major diameter AC, 8 [= 2a], and minor diameter BD, 5 [= 2b]: the area A is found initially from the relation: $\log A = \log 8 + \log 5 + \text{Difference } B$. Note: this is equivalent to $A = \pi ab = 10\pi$ in modern terms, where Difference $B = \log(\pi/4)$ is a negative amount.

Or secondly, the area of the ellipse is the mean proportional between the areas of the circles with the major and minor diameters: $2 \log A = (\log 8^2 + \text{Difference } B) + (\log 5^2 + \text{Difference } B)$; [equivalent to $\pi a^2/\pi ab = \pi ab/\pi b^2$].

Finally, the minor axis is to the major axis as the area of the ellipse is to the area of the minor circle: $\log A = (\log 5^2 + \text{Difference } B) + \log 8 - \log 5$; [equivalent to $A = \pi ab = \pi b^2 \times a/b$].

Note: in Table 26-3, Briggs prefers to add $\log 2$ rather than subtract $\log 5$ in the last method, thus using complementary arithmetic to avoid dealing with negative logarithms.

§2. Another method is presented for finding the area A of the ellipse. The difference in the areas of two concentric circles A1 and A2 (> A1), is equal to the area of an ellipse which has major and minor axes equal to the sum and difference of the diameters 8 and 5 of the circles, :

$\log A1 = (\log 5^2 + \text{Difference } B)$; $\log A2 = (\log 8^2 + \text{Difference } B)$; An ellipse is taken with axes 3 and 13, for $8^2 - 5^2 = 39$, has area $A = A2 - A1$, while $\log A = \log 3 + \log 13 + \text{Difference } B$.

[This is equivalent to $\pi(a^2 - b^2) = \pi(a - b)(a + b)$].

§3. The surface area and volume of a right cylinder with height D equal to its diameter are compared in turn with the surface area and volume of the circumscribed cube and inscribed sphere of the same width. The respective reduced ratios for both the areas and volumes of the cube, cylinder, and sphere in this order will be :- diameter : (circumference of great circle)/4 : (circumference of great circle)/6.

[This is equivalent to $1 : \pi/4 : \pi/6$. The areas are then in the ratio $6D^2 \times (1 : \pi/4 : \pi/6)$, while similarly the volumes are $D^3 \times (1 : \pi/4 : \pi/6)$].

The oblate spheroid is generated by rotating the semi-ellipse of the ellipse about the shorter axis, and the prolate spheroid similarly about the longer axis. See Fig.26-1. The volumes of the spheroids are mean proportionals between the volumes of the large and small spheres associated with the major and minor axes

[according to: $\pi/6 \cdot (2a)^3 > \pi/6 \cdot (2a)^2 \cdot (2b) > \pi/6 \cdot (2a) \cdot (2b)^2 > \pi/6 \cdot (2b)^3$].

Briggs finds the logarithm of the volume of the spheroid by first evaluating the logarithm of the volume of the parallelepiped associated with the three axes; the logarithm of the volume of the circumscribed cylinder follows by taking Difference B [i.e. the logarithm of $\pi/4 \cdot (2a)^2 \cdot (2b)$ or $\pi/4 \cdot (2a) \cdot (2b)^2$ is found]: the logarithm of the volume of the spheroid then follows by taking away the logarithm of $1^{1/2}$ [i.e. corresponding to $2/3 \times \pi/4 \cdot (2a)^2 \cdot (2b) = \pi/6 \cdot (2a)^2 \cdot (2b)$, etc].

The volume of the spheroid V_{SD} , taken here as oblate, can also be found by proportion from the volume of the sphere V_S that shares a diameter a with the spheroid, according to $V_{SD}/V_S = 2b/2a$; or directly from the parallelepiped by taking Difference D [i.e. $\pi/6 \cdot (2a) \cdot (2b)^2$ or $\pi/6 \cdot (2a)^2 \cdot (2b)$ directly].

§4. The volume of the section of a spheroid. According to Archimedes, for the prolate spheroid with the longer axis $2a$, if this axis is cut by a perpendicular vertical plane to give a short segment of length d , and a long segment of length f , so that $f + d = 2a$, (see Fig. 26-5 in the Notes), then:

(Volume of smaller spheroid segment)/($f + a$) = (Volume of smaller cone)/ f ; and similarly

(Volume of larger spheroid segment)/($d + a$) = (Volume of larger cone)/ d ; where the cones have the same base and height as their respective sections.

Briggs establishes the diameter of the prolate segment from proportions associated with the ellipse and its great circle, for given a , b , d and f . Subsequently, the area of the base circle and the volume of the cone are found; and from the above theorem, the volume of the segment of the spheroid found. The volume of the other segment of the spheroid is calculated in a similar way, and their sum compared with the volume of the spheroid, calculated separately. A similar set of calculations is performed for the oblate spheroid.

Briggs then considers the case of the spheroid with two sections, as an approximation to a wine cask. The volume of the prolate spheroid associated with the cask of given dimensions is found, and the volumes of the end segments calculated and removed. Another method for finding the cask volume involves averaging the area of cross-section: the area of the inner circle is taken with two thirds of the area of the ellipse with axes formed from the sum and difference of the max. and min. width of the cask; this area is multiplied by the height of the cask.

§26.2

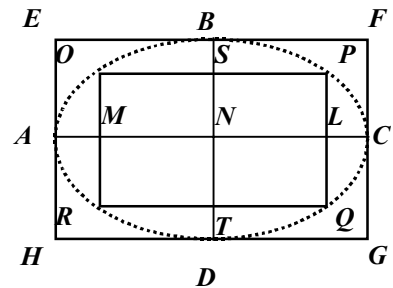
Chapter Twenty Six. [p.69]

Concerning the Ellipse, Spheroid, and Cask.

1.

The ellipse is the common intersection of the surface of a cone & the surface of a plane, cutting the whole cone. Furthermore, taking this figure for the ellipse itself; a section of this kind may be considered to be an elongated circle or an oval shape; for in the works of Archimedes spheroids are discussed which hold a likeness to the sphere, but with a shape comprised of unequal distances from the centre. For the ellipse ABCD, either the periphery going around is described, or the inside [i.e. the area] of the figure. The longest diameter is AC, the shortest BD, the former intersecting at right angles in the centre E, [See Fig. 26-1].

The ellipse is the mean proportional [area-wise] between the circles with the diameters AC, BD; as shown in the work of Archimedes: On Conoids and Spheroid, Prop.5; for the oblong rectangle, with the unequal sides of the squares described, is the mean proportional between the squares: thus the area of the ellipse is the mean proportional between the areas of the circles of the unequal diameters. And the rectangle described by the diameters AC, BD, is to the ellipse: as the square of the diameter is to the circle.



[Figure 26-1]

Therefore, given the diameters AC, BD; we can hence find the area of the ellipse. From the logarithms of the diameters the difference B is taken [i.e. log(π/4)], (which from the above chapter, taken from the logarithm of the square, leaves the logarithm of the area of the circle) the remainder is the area of the ellipse. Let the diameters be AC, 8: BD, 5.

		<i>Logarithms</i>	
Diameters	{	8	090308999
		5	06989700
Sum			160205999
Difference B			010491012
Logarithm of the area			149714987
			Area of Rectangle 40 formed from the
			Diameters
			Area of the ellipse 3141592692.

[Table 26-1].

The same area of which the logarithm is sought may be found, if in place of taking the difference *B*, its complement is added. As we have shown in Chapter 15, thus
0.]

		<i>Logarithms</i>	
Diameters	{ 8	090308999	
	{ 5	06989700	
	Sum	160205999	
Complement of difference <i>B</i>		<u>989508988</u>	
	(1)149714987		Log. of the area [Table 26-2]

The same area is produced, if the mean proportional is sought as per Ch.17, between the given diameters of the circles, found as by Ch. 25.

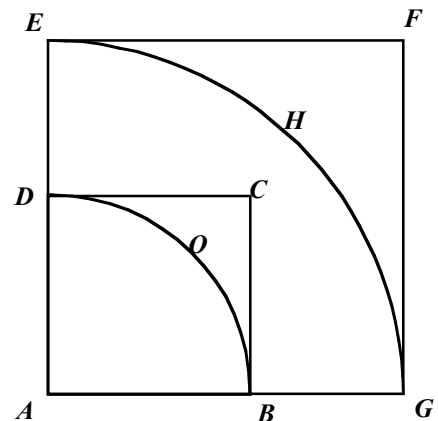
		<i>Logarithms</i>	
Square of the diameter AC	64	180617997	
Difference <i>B</i> taken in both places		010491012	
Square of the diameter BD	25	<u>139794001</u>	
Remaining log. of area of major circle		170126985	Area of major circle 50 <u>2654824</u>
Remaining log. of area of minor circle		<u>129302989</u>	Area of minor circle 19 <u>6349541</u>
Sum of the logs		299429974	
Log. of mean proportional		149714987	Area of ellipse - - -3 <u>141592692</u>

Proportions		<i>Logarithms</i>
{	or, as the minor diameter 5	930102999
	to the major diameter 8	090308999
	thus the area of the minor circle	<u>129302989</u>
	to the ellipse	1149714987

[Table 26-3]

2. If two circles are concentric, the area lying between the peripheries of these is equal to the area of the ellipse, the diameters of which ellipse are the sum and difference of the given diameters of the circles.

For as with squares, the difference of the squares is equal to the rectangle described by the sum and difference of the sides: thus for circles, the difference of the areas of the circles is equal to the area of the ellipse, of which the diameters are equal to the sum and difference of the given diameters. For the rectangle taken from the diameters is the mean proportional between the squares of



[Figure 26-2]

the diameters: & the area of the ellipse is the mean proportional between the areas of the circles of these diameters.

Therefore [area-wise], the square will be to the rectangle as the circle to the ellipse, and conversely; as the square to the circle, so the rectangle to the ellipse: And the rectangle, which is equal to the difference of the squares to the ellipse, which is equal to the difference of the area of the squares to the circles.

	30 <u>6305284</u> ellipse. The difference.	
rectangle 39. The difference.		
square 64 square 25	circle 50 <u>2654824</u>	circle 19 <u>6349541</u>

[Table 26-4]

Let the longer diameter be 8, the shorter 5. The squares are as 64: 25. The circles 502654824 : 196349541. The difference of the squares 39. The difference of the circles 306305284.

Let the sum of the diameters of the ellipse be 13, and 3 the difference of the given diameters. The rectangle is 39, of which the logarithm is 159106461. The difference **B** taken away, leaves 148615448, the logarithm of the area of the ellipse 306305284.

		<i>Logarithms</i>	
Diameters	}	13	111394335
		3	047712125
Complement of difference B			<u>989508988</u>
Log. of the diff. of the circles			1148615448 Area of the ellipse 30 <u>6305284</u> .

[Table 26-5].

3. If for a cylinder, in which a sphere is inscribed, a cube is circumscribed: the quantities¹ involved [in the order Cube : Cylinder : Sphere] are as the diameter of the sphere [D]; to the quarter of the circumference of the great circle [$\frac{1}{4}\pi D$]; & as the sixth part of the same [$\frac{1}{6}\pi D$]. Thus the square of the diameter [D^2], to the area of the great circle [$\frac{1}{4}\pi D^2$], & as two thirds of the area of the same circle [$\frac{2}{3}(\pi D^2/4) = \frac{1}{6}\pi D^2$]. And thus the surface of the cube [$6D^2$], to the surface of the cylinder [$6 \times \pi D^2/4$], & the surface of the sphere [$6 \times \pi D^2/6$]. Thus also the volume of the cube [D^3], to the volume of the cylinder [$\frac{1}{4}\pi D^3$] & the volume of the sphere [$\frac{1}{6}\pi D^3$].

Let the Diameter of the Sphere be 7 .

A Quarter of the Circumference is

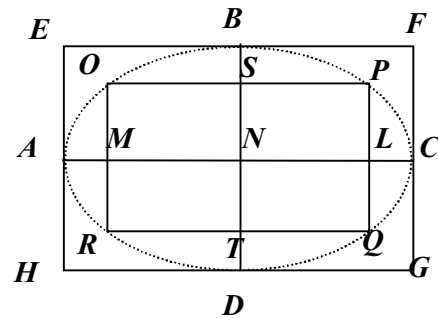
54977871438 The first three ratios with the
36651914292 remaining ratios following in
 proportion.

A Sixth of the Circumference

].71.]	Square of the Diameter	49
	(Area of) Circle	<u>38484845100066</u>
	² / ₃ of Circle	<u>256563400044</u>
	Surface of the Cube	294
	Surface of the Cylinder	<u>2309070600396</u>
	Surface of the Sphere	<u>1539380400264</u>
	Volume of Cube	343
	Volume of Cylinder	<u>2693915700462</u>
	Volume of Sphere	<u>1795943800308</u>

[Table 26-6].

The spheroid is made by the rotation of half the ellipse by keeping other axis fixed. If the longer diameter AC remains, the spheroid is made long [*i.e.* prolate]; but if BD remains, the spheroid becomes broad [oblate], of which both are means in continued proportion, between spheres



[Figure 26-3]

of unequal diameters: & each of these is in the ratio of two to three to the cylinder, of which the altitude is equal to the remaining diameter, the base is still circular, having been described by the movement of the semi-diameter turning in a circle².

With the height and width of a spheroid given, the volume of which we can thus find: The logarithm of the height is added to the logarithm of the square of the thickness: the total is the logarithm of the parallelepiped of the same altitude and base of the square. Hence, the difference **B** of Chapter 25 is taken away. There remains the logarithm of the cylinder circumscribing the spheroid: from which if the difference of the ratio of one and a half [*i.e.* $\log 3/2$] is taken away 017609125905568, there remains the logarithm of the spheroid. For, let the diameters of the ellipse be 8, 5. & by keeping the small diameter the spheroid becomes wide: the volume is sought³.

	Terms	Logarithms
of the ratio of one & half {	} 3	047712125471966
		030102999566398
	Diff. of logs.	017609125905568

	<i>Logarithms</i>
Wide diameter 8	090308999
Square 64	180617997
Height 5	<u>069897000</u>
Vol. of parallelepiped 320	250514997
Difference B being taken	<u>010491012</u>
Cylinder 251 <u>32741</u>	240023985
Difference of the ratios by one & half	<u>017609126</u>
Vol. of spheroid 167 <u>551605</u>	222414859

[Table 26-6]

Or, by Ch. 25, with the logarithms of the square & height, is added to the complement of the difference B, & the difference of the ratio of one and a half: the total (taking away the first digit 2) is the logarithm of the spheroid.

	<i>Logarithms</i>
Square 64	180617997
Height 5	069897000
Comp. Diff. B	989508988
Compl. Diff. ratio one & a half	<u>982390874</u>
Volume of spheroid	2222414859

[Table 26-7]

We find the same volume of the spheroids by the rule of proportion. For the parallelepipeds of which the bases are equal are in proportion with their own heights. In the same way as cylinders, so with spheroids inscribed in cylinders: if they have the same width, they are in proportion with heights. There are, therefore, a sphere to a spheroid of the same width; as the diameter or height of the sphere to the height of the spheroid. Therefore the logarithm of the sphere for the given diameter can be found, per Ch. 25, etc.

2.]

	<i>Logarithms</i>
Diameter 8	090308999 1
Cube of the diameter	270926996 3
Difference B taken away	<u>028100138</u>
Volume of sphere	242826858

Proportions		<i>Logarithms</i>
{	Height of sphere 8	909691001
	Height of spheroid 5	069897000
	Volume of sphere	<u>242826858</u>
	Volume of spheroid	2222414859

[Table 26-8]

Or because the parallelepiped is to the volume of the inscribed spheroid, as the cube to the sphere: from the logarithm of the parallelepiped first found, the difference *D* is taken away. Ch. 25. There is left the logarithm of the spheroid.

passing through PQ, & perpendicular to the axis AC: I assert the segment of the spheroid PCQ, to be to the cone, of which the base is the circle of diameter PQ, with the height CL: as the sum of NA & AL, to AL. Let AC, 20; BD, 12; CL, 2 ; PL is $3^{3/5}$. For if two arcs from the ellipse ABCD are described, the radii of which are equal to the lines NC, NB. & PL, PVS are drawn perpendicular to the radii NC, NB; then NZ, NB : LK, LP are proportional; likewise NC, NX: SP, SV. But LK by Prop. 13, Book 6, Euclid is the mean proportional between CL, 2 & LA, 18 [i.e. $LK^2 = AL.LC$]; therefore LK is 6, and LP 36
 [as $LP = (NB/NZ).LK$].

	proportion $\left\{ \begin{array}{l} NZ \quad 10 \\ NB \quad 6 \\ LK \quad 6 \\ LP \quad 36 \end{array} \right.$	proportion $\left\{ \begin{array}{l} AL \quad 18 \\ LK \quad 6 \\ LC \quad 2 \end{array} \right.$	
<i>Logarithms</i>			
Diameter PQ	72		085733250
Square of PQ			171466500
Difference B taken away			<u>010491012</u>
Area of circle PQ	407150408		160975488
Altitude CL	2	814300816	<u>030102999</u>
Cylinder <i>PbdQ</i>			191078487
Log. of 3 taken away			<u>047712125</u>
Cone 1/3 of cylinder PCQ	271433605		<u>143366362</u>
<i>Logarithms</i>			
pro- port- -ion.	$\left\{ \begin{array}{l} AL \quad 18 \\ NA + AL \quad 28 \\ \text{Cone } PCQ \\ \text{Seg. of spheroid } PCQ \end{array} \right.$	Compl. Arith. 271433605 422230053	874472749 144715803 <u>143366362</u> 1162554914
<i>Logarithms</i>			
Circle <i>PL</i>	407150408		160975488
Altitude <i>AL</i>	18		<u>125527251</u>
Cylinder <i>mPQn</i>	7328707343		286502739
log. of 3 taken away			<u>047712125</u>
Cone <i>APQ</i> 1/3 of cylinder	2442902447		<u>238790614</u>
pro- port- -ion.	$\left\{ \begin{array}{l} CL \quad 2 \\ CN + CL \quad 12 \\ \text{Cone } PAQ \\ \text{Seg. of spheroid } PAQ \end{array} \right.$	Compl. Arith. 2442902447 14657414684	969897001 107918125 <u>238790614</u> 1316605740
<i>Logarithms</i>			
		Segment PCQ	422230053
		Segment PAQ	14657414684
		Whole spheroid	15079644737
<i>Logarithms</i>			
Diameter BD	12		107918125
Square of BD			215836249
Difference B taken away			<u>010491012</u>
Circle BD		1130973355	205345237
Axis AC	20		<u>130102999</u>
Cylinder		2261946710	335448236
Difference of half the ratio			<u>017609126</u>
Spheroid		15079644737	317839110

[Table 26-10]

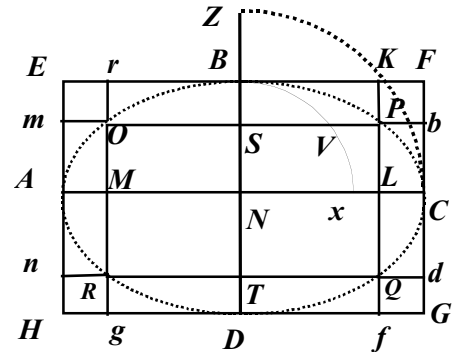
74.] But if BD is the oblate axis of the spheroid, & crosses the plane cutting the line OP : OP are of 16 parts, & the circle of which the diameter OP is 2010619298297472 ; SB , 24; SD , 96. The cylinder $rOPK$ is 48254863159 . The cone OBP is 16084954386 .

		Logarithms
Diameter OP	16	120411998
Square of OP	256	240823996
Difference B taken away		<u>010491012</u>
Area of circle OP	20106192983	230332985
Altitude BS	24	<u>038021124</u>
Cylinder $rKPO$		268354109
Log. of 3 taken away		<u>047712125</u>
Cone OBP	16084954386	220641984
<hr/>		
pro- port- -ion.		Logarithms
$\left\{ \begin{array}{l} SD\ 96 \\ ND + SD\ 156 \\ Cone\ OBP \\ Segment\ OBP \end{array} \right.$	compl. arith.	801772877
		219312460
		<u>220641984</u>
		26138030877
		1241727321
<hr/>		
Circle OP	20106192983	230332985
Altitude SD	96	<u>098227123</u>
Cylinder OgP		193019452637
log. of 3 taken away		047712125
Cone ODP	64339817546	280847983
<hr/>		
proportions.		Logarithms
$\left\{ \begin{array}{l} BS\ 24 \\ BS + BN\ 84 \\ Cone\ ODP \\ Segment\ OADCP \end{array} \right.$	Compl. Arith.	861978876
		192427929
		<u>280847983</u>
		<u>22518936141</u>
		1335254788
<hr/>		
		Segment $OADCP$
		<u>22518936141</u>
		Segment OBP
		<u>26138030877</u>
		Whole spheroid
		<u>25132741229</u>
<hr/>		
		Logarithms
Diameter AC	20	13010299956
Cube of AC	8000	39030899870
Difference D taken away		<u>02810013777</u>
Sphere	41887902047864	36220886093
<hr/>		
Proportions.		Logarithms
$\left\{ \begin{array}{l} AC\ 20 \\ BD\ 12 \\ Sphere\ of\ diameter\ AC \\ Spheroid\ BADC \end{array} \right.$	Compl. Arith.	86989700044
		10791812460
		<u>36220886093</u>
		<u>25132741228718</u>
		134002398597

[Table 26-11].

And in this way if a spheroid is cut by a single plane, perpendicular to the axis, we can find the volume of both segments.

There remains the segment that is described by the surface of the spheroid, & two planes perpendicular to the axis and equidistant from the centre. It is our task of whatever kind, the capacity of which we can measure, following that which was said above from Archimedes. Thus



[Figure 26-4]

with Book 3, Ch. 10 of *Pantometria*, that learned book in the vernacular by T.D. [Thomas Digges], the most distinguished of men, is conscripted. Also Errardus *Barleduc*, Book 3, Ch. 10, and Clavius, *Geom. Practicae*, Book 5, Ch. 10.

Let the cask be *BPQDRO* of which the height is *ML* of 21 parts, while the width of the middle is *BD*, 14. The diameter of the base or the width of the end is *PQ* $9\frac{1}{3}$. To begin with the whole length of the spheroid *AC* is required, which we find thus: *PQ* $9\frac{1}{3}$ is taken from *BD* 14, and half the remainder $4\frac{2}{3}$ is *BS* $2\frac{1}{3}$. Again *SD* is $11\frac{1}{3}$, & *SV* the mean proportional between *BS* & *SD* [*i.e.* again we use this useful theorem, to give $SV^2 = BS \cdot SD$], is $\underline{.27\frac{1}{9}}$, by Euclid, Book 6, Prop. 13, or 75.] 52174919477. But *SV*, *SP*: *NX* (or *NB*), *NC* are proportional [*i.e.* $SV/SP = NX/NC$]. *NC* therefore is $\underline{.19845}$ or 1487228257 & LC 3587228357. The circle with diameter *PQ* has the area 6841690667819. The cylinder *PbdQ* has the volume 24542706089, the cone *PCQ* has the volume 81809020297. With the two segments *PCQ*, *OAR* of volume 25736283611, taken from the spheroid, leaves the cask with volume *BPQDRO*. But if the circle of diameter *BD* 15393804400259, multiplied by the line *AC* 28174456514 makes 433712061456 for the volume of the cylinder *EFGH*, & the whole of the spheroid is 2891413743. From which the two segments 2573628361 taken will leave the volume of the cask 2643050907.

pro-	{	AL	<u>24587228257</u>
port-		AL + AN	<u>38674456514</u>
ion.		Cone	<u>81809020297</u>
		Segment	<u>12868141805</u>

[Table 26-12]

We come upon the volume of the same cask much more easily in the following way. The circle *DB* is sought with the mean width 14, & the base *PQ* $9\frac{1}{3}$. These circles are multiplied by the altitude *ML* 21: the products is the cylinders *rKfg* & *OPQR*. Taking then the difference of these cylinders, of which the difference $\frac{1}{3}$ is taken from the greater cylinder, or $\frac{2}{3}$ is added to the lesser cylinder gives the volume of the cask.

BD Diameter 14	<i>Logarithms</i>
The Square 196	229225607
Difference <i>B</i> taken away	010491012
Circle BD <u>1539380400</u>	<u>218734595</u>
PQ $9\frac{1}{3}$	
Square of PQ	19400735532
Difference <i>B</i> taken away	01049101186
Circle PQ <u>68416906678</u>	<u>18351634346</u>
Large cylinder <u>3232698840</u>	
Small cylinder <u>1436755040</u>	
Difference <u>1795943800</u>	
$\frac{1}{3}$ of difference <u>598647933</u>	
Cask <u>2634050907</u>	

[Table 26-13]

Or the area of the circle *PQ* is taken, & the ellipse of which the diameters are equal to the sum of the lines *BD*, *PQ* $23\frac{1}{3}$, & two thirds of the difference of the same $3\frac{1}{9}$; let this sum be multiplied by the given altitude *ML* 21: the product is equal to the volume of the cask.

Circle PQ <u>68416906678</u>	<i>Logarithms</i>
Sum of <i>BD</i> , <i>PQ</i> $23\frac{1}{3}$	13679767853
$\frac{2}{3}$ Difference <i>BD</i> , <i>PQ</i> $3\frac{1}{9}$	04929155219
Circle <i>BD</i> <u>1539380400</u>	<u>218734595</u>
Complement difference <i>B</i>	98950898814
Ellipse <u>570140889</u>	<u>19400735532</u>
Circle + ellipse <u>1254309956</u>	<u>01049101186</u>
Cask <u>2634050907</u>	

[Table 26-14]

The same volume as before has been found by this method⁵. Here is the reason for this procedure. The difference of the areas of the circles of the middle width *BD*, & of the ends *PQ*, is equal to the area of an ellipse, the diameters of which is equal to the sum & difference of the diameters *BD* & *PQ*, as I have shown in section 2 of this chapter. This is really the difference of the areas of the concentric Circles, that is, the base of the hollow cylinder, of which the third part is placed between the surfaces of the exterior cylinder and the Spheroid, the remaining two thirds are within the same spheroid. The area of the ellipse therefore which is added to the area of the small circle amounts to two thirds of the difference of the areas of the circles *BD*, *PQ* (for the whole difference of the diameters is not taken, but $\frac{2}{3}$ of the same $3\frac{1}{9}$; because if I take the whole difference & as much as $\frac{2}{3}$ of the sum instead, the same ellipse comes out) which together with the

§26.3

Notes On Chapter Twenty Six

¹ The ratios are taken in the order cube : cylinder : sphere. The original ratios given are :-

$1 : \pi/4 : \pi/6$, where we have succumbed to modern usage with π . For a diameter D , the area of the cube is $6D^2$, the cylinder has a surface area, including the ends, of $3\pi D^2/2$; while the sphere has surface area πD^2 . These areas can also be put in the ratio $1 : \pi/4 : \pi/6$.

Similarly, the volumes are in the same ratio $1 : \pi/4 : \pi/6$. This is Briggs' rule of proportions.

² For if m_1 and m_2 are the two means sought, starting from the larger, then we require:

$$\frac{(2a)^3}{m_1} = \frac{m_1}{m_2} = \frac{m_2}{(2b)^3}. \text{ From which it follows that } m_1 = (2a)^2(2b) \text{ and } m_2 = (2a).(2b)^2.$$

³ The volume of the ellipsoid with axis $2a, 2b, 2c$ is $\pi(2a)(2b)(2c)/6$, where $a > b > c$. If two of the axis are equal, then we have the volumes of the Spheroids: $V_1 = \pi(2a)^2(2c)/6$, if $b = c$; and $V_2 = \pi(2a)(2b)^2/6$, if $b = c$.

In the first case, the circumscribing Parallelepiped has volume $(2a)(2b)^2$, while the circumscribed cylinder has volume $\pi(2a)(2b)^2/4$. Thus, Briggs evaluates in order, the volumes $1.(2a)(2b)^2$, $(\pi/4)(2a)(2b)^2$, and $(\pi/6)(2a)(2b)^2$. Briggs subsequently finds V_2 in a like manner.

⁴ Proposition 31,32 On Conoids and Spheroids,

Archimedes, may be stated, according to Figure 26-5,

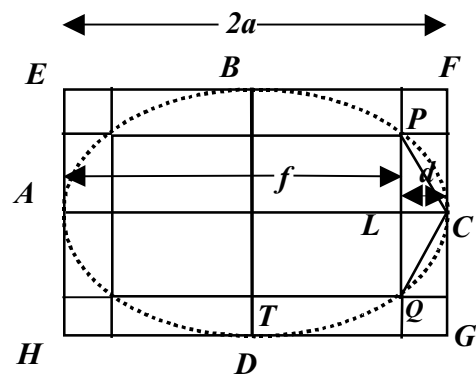
in the form, where $AC = 2a$:-

$$\text{(Volume of smaller spheroid segment PCQ)/(} f + a \text{)}$$

$$= \text{(Volume of cone PCQ)/} f; \text{ and}$$

$$\text{(Volume of larger spheroid segment PAQ)/(} d + a \text{)}$$

$$= \text{(Volume of cone PAQ)/} d.$$



[Figure 26-5]

To check the results, we may write the volume of the small segment of the spheroid as

$$V_R = (\pi/3)LP^2d(1 + a/f); \text{ and similarly}$$

$V_L = (\pi/3)LP^2f(1 + a/d)$ for the large segment. Subsequently, the total volume

$V = (\pi/3)LP^2[d(1 + a/f) + f(1 + a/d)] = (\pi/6)LP^2(2a)^3/fd$. But as in the text, $LP/LK = 2b/2a$, and $LP^2 = (2b)^2/(2a)^2 \cdot fd$ hence $V = (\pi/6)(2b)^2(2a)$ as required.

Briggs leans on the work of Archimedes for this application of Logarithms. We may recall that the treatise *The Method*...., in which Archimedes disclosed the method by which he had discovered his wonderful results, lay undiscovered at this time. The Archimedes' Palimpsest is again a centre of active research on its re-emergence, after the detective work and genius of Heiberg in its discovery and the production of a translation under difficult circumstances, and its subsequent disappearance.

[See, e.g. *Physics Today*, Volume 53, No. 6; June 2000, '*The Origins of Mathematical Physics: New Light on an Old Question*', by Reviel Netz].

⁵ The area of the ellipse in question is $(\pi/4)(BD^2 - PQ^2) = (\pi/4)(2b - PQ)(2b + PQ)$. Now, $2/3$ of the area of this ellipse is placed outside the circle with diameter PQ, giving a total area of:

$\pi \cdot LP^2 + (1/6)\pi \cdot (BD^2 - 4LP^2) = (1/6)\pi \cdot (BD^2) + (1/3)\pi \cdot (LP^2) = (1/6)\pi \cdot (2b)^2 + (1/3)\pi \cdot (2b/2a)^2 \cdot fd$, and

the volume of the cask $V_C = (\pi/6) \cdot (4b^2)[1 + fd/2a^2] \cdot 2(a - d)$, as $PQ^2 = 4LP^2 = 4(2b/2a)^2 \cdot fd$; and

$BD = 2b$. We may subsequently write $V_C = (\pi/6) \cdot (2b)^2 [2a - d^2/a - fd^2/a^2]$.

We have to reconcile this formula for V_C with that obtained above:–

The volume of the entire spheroid is $(\pi/6)ab^2$, Hence, the volume of the cask V_C is given by:

$V_C = (\pi/6)(2a)(2b)^2 - (\pi/3)(2b/2a)^2 \cdot fd \cdot 2d(1 + a/2) = (\pi/6)(2b)^2 [2a - fd^2/a^2 - d^2/a]$, as above.

§26.4.

Caput XXVI. [p.69.]

De Elleipsi, Sphaeroide, & Dolio.

Elleipsis est communis intersectio superficiei conicae & superficiei planae, secantis conum ex omni parte. Sumitur etiam elleipsis pro ipsa figura, ab huiusmodi sectione comprehensa; quae dici poterit circulus oblongus, vel Cycloides: ut apud Archimedes sphaeroeides dicitur, quae sphaerae similitudinem obtinens, inaequaliter distat a medio comprehensi spatij. ut ABCD Elleipsis, est vel peripheria ambiens, vel figura intus comprehensa. eius diameter longissima AC, brevissima BD, priorem ad rectos angulos intersecans in Centro E.

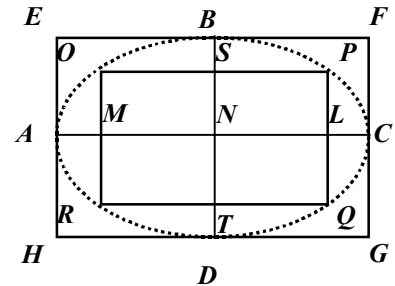
Elleipsis est media proportionalis inter circulos Diametrorum AC, BD; *Archim. prop 5.lib.de Conoid.* nam ut Oblongo rectangulum, a lateribus inaequalium quadratorum comprehensum, est medium proportionale inter Quadrata: sic Elleipsis est media proportionalis inter Circulos diametrorum inaequalium. Et rectangulum a Diametris AC, BD comprehensum, est ad Elleipsim: ut Quadratum Diametri ad Circulum. Datis idcirco Diametris AC, BD; area elleipsis

sic inuenimus. A Logarithmis Diametrorum auferatur Differentia **B**, (quae superiori Capite ablata e Logarithmo Quadrati relinquebat Logarithmum circuli) reliquus erit Logarithmus Elleipsis. ut sunt Diametri AC, 8; BD, 5.

	<i>Logarithmi.</i>		
Diametri	{ 8	090308999	
	{ 5	06989700	
Summa		160205999	ectanguli 40 comprehensi a diametris.
Diff. B		010491012	
Logarithmus areae		149714987	area Elleipsis 3141592692.

Idem Areae quaesitae Logarithmus inuenietur, si loco Differentiae B, auferendae, additur eius complementum. ut in Cap. 15 ostendimus, ut

	<i>Logarithmi.</i>		
Diametri	{ 8	090308999	
	{ 5	06989700	
Sum		160205999	
Compl. Diff. B		989508988	
(1)149714987			Logarith. areae



Eadem area prodibit, si quaeratur per cap.17 medius proportionalis inter datarum diametrorum Circulos, inventos per cap. 25.

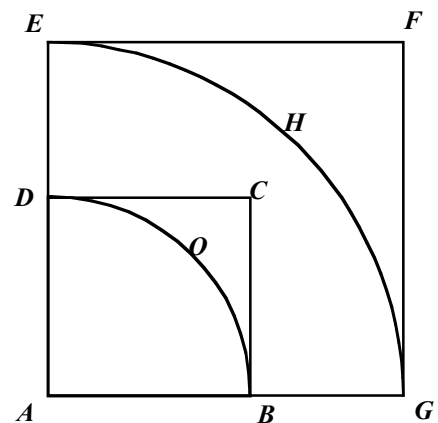
[p.70.]

	<i>Logarithmi.</i>		
Quadratum diametri AC.64		180617997	
Differentia B auferenda ab utroque		010491012	
Quadratum diametri BD.25		139794001	
Reliquus Logar. circuli maioris		170126985	circulus maior 502654824
Reliquus Logar. circuli minoris		129302989	circulus minor 196349541
Summae Logarithmorum		299429974	
Logarithmus medij proportionalis		149714987	Elleipsis - - -3141592692
Proportionem			<i>Logarithmi.</i>
{	vel, ut Diameter minor5	8	Compl. Arith. 930102999
	ad Diametrum maiorem		090308999
	sic circulus minor		129302989
	ad Elleipsim		1149714987

2. Si duo circuli sint concentrici, Area eorum peripherij interiecta aequatur Elleipsi, cuius Diametri sunt summae & differentia datarum Diametrorum.

Nam ut in Quadratis, Differentia Quadratorum aequatur rectangulo comprehenso a summa & differentia Laterum: sic in Circulis, Differentia Circularum aequatur Elleipsi, cuius Diametri aequantur summae & differentia datarum diametris. Est enim Rectangulum a Diametria comprehensum, medium proportionale inter Quadrata diametrorum : & Elleipsis est media proportionalis inter Circulos diametrorum.

Erit igitur ut Quadratum ad Rectangulum; sic Circulus ad Elleipsim, & alterne; ut Quadratum ad Circulum, sic rectangulum ad Elleipsim: & Rectangulum quod differentiae Quadratorum est aequale; ad Elleipsim, quae differentiae Circularum aequabitur.



Rectang. 39. Differentia		306305284	Elleipsis Differentia.
Quad. 64	Quad. 25	Circulus 502654824	Circulus 196349541

Ut esto Diameter longior 8, minor 5. erunt Quadrata 64. 25. Circuli 502654824 : 196349541. Differentia Quadratorum 39. Differentia circuli 306305284.

Sunto Diametri Elleipsis, 13 summa, 3 differentia diametrorum datarum. Rectangulum 39, eius Logarithmus 159106461. Differentia **B** ablata, relinquit 148615448. Logarithmum Elleipsis 306305284.

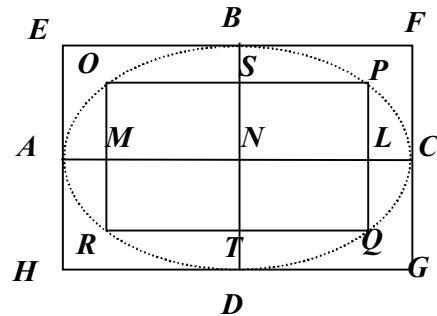
		Logarithmi	
Diametri	} 13	111394335	
		3	047712125
Compl. Diff. B		<u>989508988</u>	
Logar. diff. circulorum		1148615448	Elleipsis <u>306305284</u> .

3. Si Cylindro, cui Sphaera inscribatur, circumscribatur Cubus: erunt ut Diameter Sphaerae, ad Quadrantem Peripheriae maximi Circuli, & ad Sextantem eiusdem: sic Quadratum Diametri, ad Circuli & ad duas tertias circuli. Et sic Superficies Cubi, ad Superficies Cylindri & Sphaerae : sic etiam Cubus, ad Cylindrum & Sphaeram.

Esto Diameter Sphaerae	7	
Erit Quadrans Peripheriae	<u>54977871438</u>	tres primi tribus proximis,
Sextans Peripheriae	<u>36651914292</u>	reliquisq; sequentibus, sunt
		proportionales.
	[p. 71.]	
Quad. Diametro	49	
Circulus	<u>38484845100066</u>	
² / ₃ Circuli	<u>256563400044</u>	
Superficies Cubi	294	
Superficies Cylindri	<u>2309070600396</u>	
Superficies Sphaerae	<u>1539380400264</u>	
Cubus	343	
Cylindrus	<u>2693915700462</u>	
Sphaerae	<u>1795943800308</u>	

Sphaeroeides sit, conversione semielleipsis manente altera Diametro. Si maneat AC diameter longior, sit Sphaeroeides oblongata; sin maneat BD, Sphaeroeides lata. quae ambae sunt mediae continue proportionales, inter Sphaeras inaequalium diametrorum: & earum utraque, est in subsesquialtera ratione ad Cylindrum, cuius altitudo aequatur Diameter manenti, basis vero est circulus, motu semidiametri circumactae descriptus.

Datis altitudine & crassitudine Sphaeroeidis, eius soliditatem sic inveniemus. Addatur Logarithmo altitudinis, Logarithmo quadratae crassitudinis: totus erit Logarithmus parallelepipedum eiusdem altitudinis & basis quadratae. Hinc, auferenda est differentia **B** capitis 25. restabit Logarithmus Cylindri, Sphaeroeidi circumscripti : quo si auferatur Differentia rationis sesquialterae 017609125905568, restabit Logarithmus Sphaeroeidis. Ut sunt Diametri Elleipsis 8. 5. & manente diametro minore fiat Sphaeroeides lata: quaeritur soliditas.



	Termini	Logarithmi.
Rationis sesquialterae	} 3	047712125471966
		030102999566398
	Diff. Logar.	<u>017609125905568</u>

	Logarithmi.
Diameter crassitudinis 8	090308999
Quadratum 64	180617997
Altitudo 5	<u>069897000</u>
Parallelepipedum 320	250514997
Differentia B auferenda	<u>010491012</u>
Cylindrus <u>25132741</u>	240023985
Differentia rationis sesquialterae	<u>017609126</u>
Sphaeroeides <u>167551605</u>	222414859

Vel, per cap. 25, Quadrati & altitudinis Logarithms, addantur complementa Differentia B, & Differentiae rationis sesquialterae: Totus (dempta prima nota 2) erit Logarithmus sphaeroeidis.

	<i>Logarithmi</i>
Quadrati 64	180617997
Altitudinis 5	069897000
Compl. Differentiae B	989508988
Compl. Differentiae rationis sesquialterae	<u>982390874</u>
Sphaeroeides	2222414859

Eandem soliditatem Sphaeroeidis, per proportionis regulam inveniemus. Sunt enim parallelepipedum quorum bases sunt aequales, ipsis altitudinibus proportionalia. eodem etiam modo tam Cylindri, quam Sphaeroeides cylindris inscriptae, si sint eiusdem crassitudinis, sunt altitudinibus proportionales. Erit igitur Sphaera, ad sphaeroeidem crassitudinis; ut Diameter vel altitudo

[p.72.]

sphaerae, ad altitudinem Sphaeroeidis. Quaerendus est idcirco Logarithmus Sphaerae pro datis Diametro, per Cap. 25, &c.

	<i>Logarithmi.</i>
Diameter 8	090308999 1
Cubus Diametri	270926996 3
Differentia B auferenda	<u>028100138</u>
Sphaere	242826858

		<i>Logarithmi.</i>
Proportiones		
{ Altitudo Sphaerae 8	Compl. Arith.	909691001
{ Altitudo Sphaeroeidis 5		069897000
{ Sphaera		<u>242826858</u>
{ Sphaeroeides		2222414859

Vel quia parallelepipedum est ad Sphaeroeidem, ut Cubus ad Sphaeram: e Logarithmo parallelepipedum prius invento, auferatur Difference *D*. cap. 25. reliquus erit Logarithmus Sphaeroeidis.

	<i>Logarithmi.</i>
Parallelepipedum	250514997
Differentia <i>D</i>	028100138
Sphaeroeides	<u>2222414859</u>

Atque haec est Sphaeroeides Lata. Sphaeroeides Oblonga eodem modo inveniatur.

Esto Diameter crassitudinis 5, Altitudo 8.

	<i>Logarithmi.</i>
Diameter 5	069897000
Cubus Diametri 125	209691001
Differentia <i>D</i>	<u>028100138</u>
Sphaera	<u>181590863</u>

		<i>Logarithmi</i>
{ proportionem		
{ Altitudo Sphaerae 5	Comp. Arith.	930102999
{ Altitudo Sphaeroeides 8		090308999
{ Sphaera		<u>181590864</u>
{ Sphaeroeides 104719755		1202002862

Atque ad hunc modum, utrasque Sphaeroeides inveniimus, quae sunt continue proportionales inter Sphaeras Diametrorum inaequalium 8 & 5.

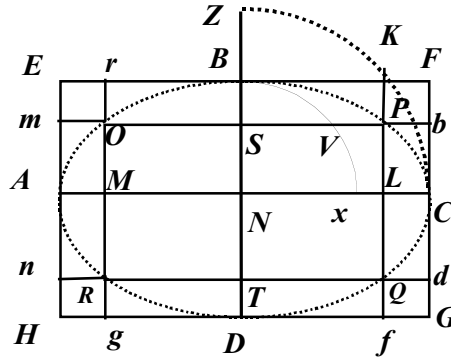
		<i>Logarithmi.</i>
Contin. propor.		
{ Sphaera cuius Diameter 8	<u>268082573</u>	242826858
{ Sphaeroeides crassa 8, alta 5	<u>167551608</u>	222414860
{ Sphaeroeides crassa 5, alta 8	<u>104719755</u>	202002862
{ Sphaera cuius Diameter 5	<u>65449847</u>	181590863

4. Si segmenti Sphaeroeidis soliditatem scire velimus, consulendum sunt Archimedis prop.31 & 33.lib. de Conoid. Quas unica hac exprimere conatus sum. Si Sphaeroeides plano secetur perpendiculari ad axem: segmentum Sphaeroeidis est ad Conum aequalium, habentem eandem cum segmento basim; ut composita ex axe dimidiato & altitudine reliqui segmenti, est ad altitudinem reliqui segmenti.

Ut esto Sphaerois ABCD, secta plano transeunte per PQ, & perpendiculari axi AC: aio segmentum PCQ, esse Conum, cuius basis est Circulus Diametri PQ, altitudo vero CLm

[p.73.]

: ut composita ex NA & AL, ad AL. Esto AC, 20; BD, 12; CL, 2 ; PL erit $3\frac{3}{5}$. Nam si in Elleipsi ABCD describantur duae peripherea quarum radij aequantur rectis NC, NB. & ducantur PL, PVS perpendicularares radij NC, NB; erunt NZ, NB : LK, LP ; item NC, NX: SP, SV proportionales. Est autem LK per 13.prop.6.lib.Eucl. media proportionalis inter CL, 2 & LA, 18, erit igitur LK 6, & LP 36



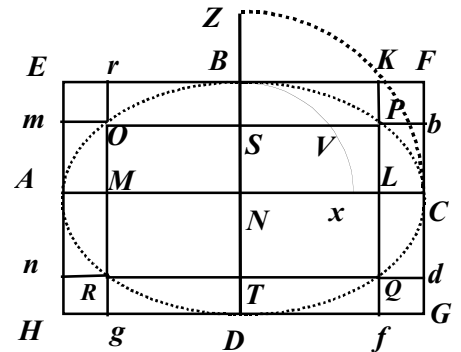
prop-	{	NZ	10	prop.	{	AL	18
port.		NB	6	LK		6	
		LK	6	LC		2	
		LP	36				

		Logarithmi.			
PQ Diameter 72		085733250			
Quadrantum PQ		171466500			
Differentia B auferenda		<u>010491012</u>			
Circulus PQ	407150408	160975488			
Altitudo CL 2	814300816	<u>030102999</u>			
Cylindrus PbdQ		191078487			
3.Logar auferendus		<u>047712125</u>			
Conus 1/3 cylindri PCQ	271433605	143366362			
		Logarithmi.			
pro-	{	AL 18	Compl. Arith.	874472749	
port.		NA + AL 28		144715803	
		Conus PCQ	271433605		<u>143366362</u>
		Segm. Sphaeidis PCQ	422230053		1162554914
		Logarithms			
Circulus PL	407150408			160975488	
Altitudo AL	18			<u>125527251</u>	
Cylindrus mPQn	7328707343			286502739	
3. Logarithmus auferendus				<u>047712125</u>	
Conus APQ ¹ / ₃ Cylindri	2442902447			238790614	
pro-	{	CL 2	Compl. Arith.	969897001	
port.		CN + CL 12		107918125	
		Cone PAQ	2442902447		<u>238790614</u>
		Segm. of sphaer. PAQ	14657414684		1316605740
		Segmentum PCQ	422230053		
		Segmentum PAQ	14657414684		
		Tota Sphaerois	15079644737		
		Logarithmi			
Diameter BD	12			107918125	
Quadratum BD				215836249	
Differentia B auferenda				<u>010491012</u>	
Circulus BD	1130973355			205345237	
Axis AC	20			<u>130102999</u>	
Cylindrus	2261946710			335448236	
Differentia rationis sesquialtera				<u>017609126</u>	
Sphaeroeides	15079644737			317839110	

Sin fuerit BD axis sphaeroeidis Lata, & transeat planum secans per rectam OP : erit OP partium 16, & Circulus cuius Diameter OP 2010619298297472; SB , 24; SD , 96. Cylindrus $rOPK$ 48254863159. Conus OBP 16084954386.

[p.74.]

		Logarithmi.
Diametri OP 16		120411998
Quadratum OP 256		240823996
Differentia B auferenda		010491012
Circulus OP	20106192983	230332985
Altitudo BS 24	48254863159	038021124
Cylindrus $rKPO$		268354109
3. Logarithmus auferendus		047712125
Conus OBP	16084954386	220641984
<hr/>		
pro- port.		Logarithmi.
$\left\{ \begin{array}{l} SD \ 96 \\ ND + SD \ 156 \\ \text{Conus } OBP \\ \text{Segmentum } OBP \end{array} \right.$	compl. arith.	801772877
		219312460
		220641984
		1241727321
<hr/>		
Circulo OP	20106192983	230332985
Altitudo SD 96		098227123
Cylindrus Og/P	193019452637	328560108
3. Logarithmus auferendus		047712125
Conus ODP	64339817546	280847983
<hr/>		
proportiones.		Logarithmi.
$\left\{ \begin{array}{l} BS \ 24 \\ BS + BN \ 84 \\ \text{Conus } ODP \\ \text{Segmentum } OADCP \end{array} \right.$	Compl. Arith.	861978876
		192427929
		280847983
		1335254788
<hr/>		
Segmentum $OADCP$	22518936141	
Segmentum OBP	26138030877	
Tota Sphaerois	25132741229	
<hr/>		
		Logarithmi
Diameter AC 20		13010299956
Cubus Diametri 8000		39030899870
Differentia D auferenda		02810013777
Sphaera	41887902047864	36220886093
<hr/>		
Proportiones.		Logarithmi
$\left\{ \begin{array}{l} AC \ 20 \\ BD \ 12 \\ \text{Sphaera Diametri } AC \\ \text{Sphaera } BADC \end{array} \right.$	Compl. Arith.	86989700044
		10791812460
		36220886093
		134002398597



Atque ad hunc modum si Sphaeroeides unico secatur plano, perpendiculari ad axem, poterimus utriusque segmenti soliditatem invenire.

Superest Segment quod superficie Sphaeroeidis, & duobus planis perpendicularibus Axi & aequidistantibus a centro comprehenditur. cuiusmodi est Dolium nostrum, cuius capacitatem metiri poterimus, secundum ea quae superius dicta sunt ex Archimede. sic Pantometria lib.3.cap.10. quem librum vir clarissimus T.D. lingua vernacula eruditissime conscripsit. sic Errardus Barleduc, lib.3.cap.10. sic Clavius Geom. Practicae lib.5.cap.10.

Ut esto Dolium $BPQDRO$ cuius altitudo sit ML partium 21, crassitudo autem media sit BD , 14. Diameter basis vel crassitudo extrema sit PQ $9\frac{1}{3}$, imprimis quaerenda est longitudo integrae sphaeroeid AC , quam sit inveniemus: PQ $9\frac{1}{3}$ auferatur BD 14, semissis reliqui $4\frac{2}{3}$ erit BS $2\frac{1}{3}$. vero erit SD $11\frac{1}{3}$, & SV media proportionalis inter BS & SD , erit $27\frac{1}{9}$, per 13.p.6.lib. Eucl. vel

[p.75.]

52174919477. sunt autem SV , SP : NX (vel NB), NC proportionales. erit igitur NC 19845 vel 1487228257 & LC 3587228357. Circulus Diametri PQ 6841690667819. Cylindrus $PbdQ$ 24542706089, Conus PCQ 81809020297. Duo segmenta PCQ , OAR 25736283611, quae ablata e Sphaeroeid, relinquunt Dolium $BPQDRO$. Est autem Circulus Diametri BD 15393804400259, qui ductus in rectam AC 28174456514 facit 433712061456, cylindrum $EFGH$ & totam Sphaeroeidem 2891413743, e qua si demantur duo segmenta 2573628361 restabit Dolium 2643050907.

pro- port.	{	AL	24587228257
		AL + AN	38674456514
		Conus	81809020297
		Segm.	12868141805

Eandem Dolij capacitatem multo facilius assequemur, ad hunc modum. Quaerantur circuli DB crassitudinis mediae 14, & basis PQ $9\frac{1}{3}$. hos circulos multiplicet altitudo ML 21: facti erunt Cylinderi rKfg & OPQR. sumatur deinde horum Cylindrorum differentia, huius differentiae $\frac{1}{3}$ ablata e Cylindro maiore, vel $\frac{2}{3}$ adiectae Cylindro minori dabunt soliditatem Dolij.

BD Diameter 14	Logarithmi.
Quadratum 196	229225607
Differentia B auferenda	010491012
Circulus BD 1539380400	218734595
<hr/>	
PQ $9\frac{1}{3}$	
Quadratum PQ	19400735532
Differentia B auferenda	01049101186
Circulus PQ 68416906678	18351634346
Cylindrus ma. 3232698840	
Cylindrus mi. 1436755040	
Differentia 1795943800	
<hr/>	
$\frac{1}{3}$ Differentiae 598647933	
Dolium 2634050907	

Vel sumatur Circulus PQ, & Elleipsis cuius diametri aequantur summae rectorum BD, PQ $23\frac{1}{3}$, & duabus tertijs differentiae earundem $3\frac{1}{9}$; horum summam multiplicet altitudo data ML 21: factus aequabitur Dolio.

Circulus PQ 68416906678	Logarithmi.
Summa BD, PQ $23\frac{1}{3}$	13679767853
$\frac{2}{3}$ Differentiae BD, PQ $3\frac{1}{9}$	04929155219
[Circulus BD 1539380400	218734595]
Compl. Differentiae B	98950898814
Elleipsis 570140889	19400735532
Circ. + Elleipsi 1254309956	01049101186
Dolium 2634050907	

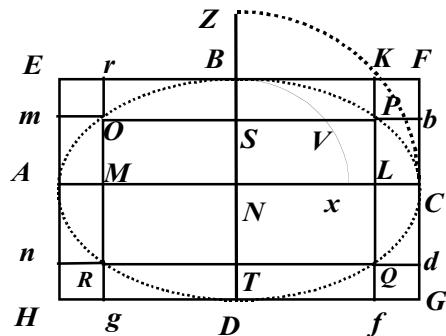
Hoc modo inventa est eadem soliditas quae antea. Cuius rei ratio haec est. Differentia Circulorum mediae crassitudinis BD, & extremae PQ, aequatur Elleipssi, cuius Diametri aequantur summae & differentiae Diametrorum BD & PQ, ut ostendi, 2. sect. huius capit. Ista vero Circulorum concentricorum differentia, est basis Cylindri Concavi, cuius pars tertia sita est inter superficies Cylindri exterioris & Sphaeroeidis, reliquae duae tertiae sunt intra Sphaeroeidem. Elleipsis idcirco quae circulo minori adijcitur valet tantum duas tertias illius differentiae Circulorum BD, PQ (sumebatur enim non integra Diametrorum differentia, sed tantum $\frac{2}{3}$ eiusdem $3\frac{1}{9}$: quod si totam differentiam & tantum $\frac{2}{3}$ summae sumpsissem, eadem Elleipsis evenisset) quae una cum Circulo

[p.76.]

minore multiplicata per datam altitudinem 21, facit eandem quae prius inventa est soliditatem Dolij.

Esto Dolium PCQRAO, cuius altitudo OR $7\frac{1}{2}$ sit minor crassitudine AC 20, & sit OP 16. Quaeritur huius Dolij capacitas. Summa Diametrorum AC + OP 36, differentia 4.

Quadratum OP 256	2408239965
Differentia B	0104910119
Circulus OP 20106192983	2303329846
<hr/>	
$\frac{2}{3}$ summae Diametrorum 24	1380211242
Differentia Diametrorum 4	0602059991
Compl. Difference B	9895089881
Elleipsis 753982235	11877361114
Circulus 2010619298	
basis 2764601533	
Altitudo $7\frac{1}{2}$	



Dolium 1990513104

Atque his modis metiri poterimus magnitudines harum figurarum, Elleipsis & Sphaeroeidis.

- 5 Si datae Elleipsi Circulum aequalem construere velimus, quaerenda est media proportionalis inter Diametros Elleipsis: Circulos cuius Diameter aequatur mediae, erit aequalis datae Elleipsi. Est enim eadem ratio Circuli ad Elleipsim, quae est Quadratum ad Oblongum. Si Sphaeroeidi Sphaeram describere velimus; inveniendae sunt duae mediae continue proportionales, inter altitudinem Sphaeroeidis & Diametrum crassitudinis. Sphaera, cuius diameter aequabitur illi mediae, quae est crassitudinis diametro propior; erit aequalis datae Sphaeroeidi. nam ita se habet Cubus ad Parallelepipedum quadratae basis, ut Sphaera ad Sphaeroeidem eiusdem cui Parallelepipedo altitudinis. cum Parallelepipedum sit ad Sphaeroeidem aequaleam, ut Quadrata basis ad $\frac{2}{3}$ Circuli.