

# Abstract

A finite group is cyclically generated if it has an automorphism that cycles through a generating set for the group. We present results that help us to determine whether a given group is cyclically generated without first identifying its automorphism group. Using these techniques we establish conditions under which certain familiar groups are cyclically generated and, in particular, we prove that every finite abelian group is cyclically generated. With the emphasis on groups that are the direct product of a cyclic group and a non abelian group, we investigate necessary and sufficient conditions for a direct product to be cyclically generated.

The question of when a finite abelian group has a minimal cyclic generating set is addressed in the second part of this thesis. As every symmetric generating set is a cyclic generating set, we can use a recent result about symmetric generating sets for abelian groups to find examples of abelian groups that have cyclic generating sets of minimum size. We explore the connection between automorphisms, matrices, and roots of certain polynomials over rings of prime power order to construct suitable automorphisms or prove that no such automorphism exists. We prove necessary and sufficient conditions for a finite abelian group of rank 3 or 4 to have a minimal cyclic generating set, and also for an abelian group of rank 5 if the order of the group is not divisible by 5. Finally, we present partial results for abelian groups of higher prime and square free rank.