Research Statement

Since completing my PhD in 2006 my research interests have developed considerably. My current interests include: combinatorial group and semigroup theory, string rewriting systems and the study of related homological finiteness properties, algebraic graph theory, infinite graph theory, homogeneous structures (in the model-theoretic sense), and groups and semigroups acting on graphs and other relational structures. I am currently involved in numerous research projects, covering these areas, and exploring connections between them. Details of some of my current projects, and future research plans, are outlined below.

Homogeneous and partially-homogeneous structures

I am interested in the study of (both finite and infinite) graphs, and other relational structures, that have a high level of symmetry. The symmetry of a structure may be measured by considering the transitivity properties satisfied by its automorphism group. The properties I am interested in lie on a spectrum ranging from relatively weak conditions, such as k-arc-transitivity and distance-transitivity, to much stronger properties like homogeneity, which is a concept arising in model theory. I am especially interested in classification problems.

Working with D. Macpherson (Leeds) we considered connected-homogeneous structures, which is a generalisation of homogeneity where one insists only that isomorphisms between connected substructures extend to automorphisms. Generalising work of Lachlan and Woodrow (1980), Schmerl (1979), and Gardiner (1978), in [9] we classified the countable infinite connected-homogeneous graphs, and posets. In recent joint work [20] with D. Macpherson (Leeds), C. E. Praeger (University of Western Australia) and G. Royle (University of Western Australia) we have considered set-homogeneous structures. Set-homogeneity is an alternative weakening of homogeneity, analogous to the weakening of k-transitive to k-homogeneous in the theory of permutation groups. We have classified the finite set-homogeneous directed graphs, generalising earlier work of Lachlan (1982). We have also classified the countably infinite digraphs that are set-homogeneous but not 2-homogeneous.

In the future in this area, in addition to considering classification problems, I am also interested in investigating the interesting connections (explored in recent work of Bodirsky and coauthors) of homogeneous structures and omega-categoricity to constraint satisfaction, a topic in complexity theory.

Groups acting on graphs and other relational structures

For infinite locally-finite graphs the notion of the ends of the graph plays an important role. Roughly speaking, the ends describe the ways that the graph "goes to infinity", and graphs with infinitely many ends may be thought of as being "tree-like". In [12] I used Dunwoody's theory of structure trees to study groups acting on infinite locally-finite graphs with more than one end. In recent joint work [5] with R. Moller (Iceland) we have used similar methods to analyse groups acting on infinite locally-finite digraphs with more than one end. This has lead to some interesting connections with the highly-arc-transitive digraphs of Cameron, Praeger and Wormald (1993).

I have investigated posets that are "tree-like" in joint work with M. Droste (Leipzig) and J. K. Truss (Leeds). Specifically we have considered, so called, cycle-free partial orders which is a concept defined in terms of the Dedekind–MacNeille completion of the poset. In [15] we considered uncountable unbalanced bipartite graphs arising from the cycle-free partial order construction, and its generalisations. Working with J. K. Truss, in [13] we developed a method for constructing countably infinite one-arc-transitive bipartite graphs, which generalises the previous construction method of Warren (1997). Also, in [10] we explored the relationship between the notion of the ends of a graph and the concept of a poset being cycle-free, by associating a certain infinite bipartite graph with the poset. In current joint work with J. K. Truss we are using order theoretic approaches developed in our previous joint work to study countably infinite locally 2-arc-transitive bipartite graphs.

In future work in this area, among other things, I would like to investigate to what extent the recently developed structure tree theory of Dunwoody and Krön (based on vertex cuts) can be used to extend results like those above to non locally finite graphs. Recent work of Hamann and coauthors shows the potential for positive results to be obtained in this direction.

Finiteness conditions for groups and semigroups

The central topic of my recent ESPRC Postdoctoral Fellowship was the study of finiteness conditions which arise in combinatorial group and semigroup theory. I am particularly interested in how the finiteness properties holding in a semigroup influence, and conversely depend on, the finiteness properties holding in the substructures of the semigroup. The finiteness properties I am interested in include computational properties like having soluble word problem, algebraic properties such as being residually finite, and various homological and homotopical finiteness properties arising from the theory of string rewriting systems. In general, a given finiteness condition will not be preserved when passing to substructures (or extensions) and so it becomes important to understand under what conditions the property will be inherited. This involves developing ways of measuring the "size" of a subsemigroup in a semigroup, in such a way that when a subsemigroup is measured to be large inside its parent, then the subsemigroup and the parent share many properties. This leads to the problem of developing a theory of "index" in semigroup theory. This is motivated by the corresponding fundamental idea in group theory, where many finiteness conditions are known to be preserved when taking finite index subgroups and extensions. A recent development in this area has been, in joint work with N. Ruskuc (St Andrews), our introduction of the concept of Green index (see [14]), which gives a much more powerful notion of index than previous attempts. In [14] and [21] (joint with A. Cain (Oporto) and N. Ruskuc (St Andrews)) we have shown that numerous important finiteness properties are preserved when taking finite Green index subsemigroups or extensions.

In collaboration with A. Malheiro (Lisbon) we have been exploring finiteness conditions arising from the theory of string rewriting systems. Specifically in we have considered finite derivation type (in the sense of Squier (1994)) for subgroups of monoids, showing the relationship between the property holding in the monoid, and it holding in its maximal subgroups [8]. During 2009, supported by a grant from the British Council (Portuguese/British), with S. J. Pride (Glasgow) and A. Malheiro (Lisbon) we investigated finite derivation type for Schützenberger groups of monoids. In [7] we show that, contrary to expectation, neither the property of being definable by a finite complete rewriting system, nor the related homotopical finiteness property finite derivation type, is inherited by a monoid from its Schützenberger groups. As an application, we use this result to show that neither of these properties is a quasi-isometry invariant of finitely generated monoids. Related to this is the study of diagram groups (in the sense of Guba and Sapir) which are fundamental groups of Squier complexes. In future work I intend to investigate how the rewriting methods used during our study of finite derivation type might be usefully used in the study of diagram groups.

Working with S. J. Pride (Glasgow), we have been investigating closure properties of the homological finiteness property FP_n. In [1] we give methods for constructing free resolutions for monoids from free resolutions of their substructures, and vice versa, and then apply these results to show how the property FP_n holding in a monoid relates to the same property holding in the ideals, and maximal subgroups, of the monoid. In group theory, it is often easier to establish the topological finiteness properties F_n for a group than the homological finiteness properties FP_n, especially if there is a suitable geometry or topological space available on which the group acts nicely. In the future I am interested in developing a corresponding theory of topological finiteness properties of monoids. Once developed, such a theory should greatly assist the study of homological finiteness properties of monoids.

Geometric group and semigroup theory

A fundamental idea in geometric group theory is that a group can be understood by studying the way it acts, in a suitably controlled way, upon a metric space. The Švarc–Milnor lemma guarantees that a group which acts in a suitably controlled way upon a geodesic metric space is quasi-isometric to that space. Quasi-isometry is a notion of equivalence between metric spaces which captures formally the intuitive idea of two spaces looking the same "when viewed from far away". An important theme in geometric group theory is Gromov's programme of classifying finitely generated groups up to quasi-isometry.

In recent joint work with M. Kambites (Manchester) we have initiated a programme with the aim of extending key techniques and concepts, like those mentioned above, from geometric group theory to semigroup theory. We began this work in [22] where we study groups acting by length-preserving transformations on spaces equipped with asymmetric, partially-defined distance functions (called semimetric spaces). We introduce a natural notion of quasi-isometry for such spaces and exhibit an extension of the Svarc–Milnor Lemma to this setting. Among the most natural examples of these spaces are finitely generated monoids and semigroups and their Cayley and Schützenberger graphs. We have applied our results to show a number of important properties of monoids are quasi-isometry invariants. In [3] we study monoids acting by isometric embeddings on spaces equipped with asymmetric, partially-defined distance functions. The canonical example of such an action is a cancellative monoid acting by translation on its Cayley graph.

In future work in this area we intend to extend the theory to higher dimensions, and in particular hope to obtain a Macbeath type theorem for monoids acting on directed spaces. I am also interested in investigating further the extent to which the algebraic properties of a finitely generated monoid are reflected in the geometric properties of its Cayley graph. Relating to this, in recent joint work with Ruskuc we have introduced the notion of the boundary of a subsemigroup of a finitely generated semigroup which, intuitively, describes the position of the subsemigroup inside the left and right Cayley graphs of its containing semigroup. Among other things, we prove in [4] that finite presentability is inherited by subsemigroups with finite boundary. In future I would also like to identify further quasi-isometry invariants of monoids, and also determine the algebraic significance of the number of ends of a finitely generated monoid.

Idempotent generated semigroups

The set of idempotents of an arbitrary semigroup has the structure of a so called biordered set (or regular biordered set in the case of von Neumann regular semigroups). These structures were studied in detail in work of Nambooripad (1979) and Easdown (1985).

There is a free object in the category of idempotent generated semigroups with a given fixed biordered set. It was conjectured by McElwee in 2002 that the maximal subgroups of a free idempotent generated semigroup on any biordered set are all free. The first counterexample to this conjecture was given by Brittenham, Margolis and Meakin (2009), where it was shown that the free abelian group of rank 2 is a maximal subgroup of the free idempotent generated semigroup arising from a certain 72-element semigroup. In recent joint work with N. Ruskuc [2] we have proved that, in fact, every group is a maximal subgroup of some free idempotent generated semigroup, and every finitely presented group is a maximal subgroup of some free idempotent generated semigroup arising from a finite semigroup.

Our plans for future research in this area include the investigation of maximal subgroups of free idempotent generated semigroups arising from natural families of semigroups. For instance, in recent joint work with N. Ruskuc [23] we have determined the maximal subgroups of free idempotent generated semigroups over the full transformation monoid.